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EDGE AND ASPECT RATIO EFFECTS ON NATURAL CONVECTION FROM THE HORIZONTAL HEATED PLATE FACING DOWNWARDS

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Abstract—Results of an experimental study of natural convection from a downward facing horizontal heated plate are reported. Measurements were made for square and rectangular plates in air, water and a high Prandtl number oil. The plates were used with bare edges and with approximately adiabatic extensions around the edges. An explanation and correlation of the edge effects is made in terms of displacing the origin of a boundary layer solution. The correlation so obtained accounts for bare edges or adiabatic extensions, and plate aspect ratio, unlike those previously available.

NOMENCLATURE

c, specific heat capacity;
C_1, C_2, C_3, C_4, empirical constants;
g, gravitational acceleration;
h, local heat transfer coefficient;
k, thermal conductivity;
L, short side length of plate;
L_a, length of adiabatic extension;
L_e, edge displacement length;
m, empirical exponent;
n, empirical exponent;
Nu_L, $\bar{h}L/K$, Nusselt number;
Nu_x, hx/k, local Nusselt number;
p, empirical exponent;
Pr, ν/α, Prandtl number;
q, heat flux;
Ra_L, $g\beta\Delta TL^3\nu^{-1}\alpha^{-1}$, Rayleigh number;
Ra_x, $g\beta\Delta Tx^3\nu^{-1}\alpha^{-1}$, local Rayleigh number;
T_w, wall temperature;
T_∞, free stream temperature;
W, long side length of plate;
x, y, z, coordinates.

Greek symbols
α, thermal diffusivity, $k/\rho c$;
β, coefficient of thermal expansion, $-(1/\rho)\partial\rho/\partial T$;
δ, boundary layer thickness;
ΔT, $T_w - T_\infty$;
ν, kinematic viscosity;
ρ, fluid density.

1. INTRODUCTION

INVESTIGATION of natural convection from the downward facing heated plate was initiated by Saunders, Fishenden and Mansion [1] and Weise [2]. Saunders, Fishenden and Mansion presented a dimensional correlation for the heat flux versus the plate-to-air temperature difference for a 46 cm × 23 cm rectangular flat plate. Weise presented results for a 16 cm square plate in air. Subsequently Fishenden and Saunders [3] presented a dimensionless correlation for square plates up to 61 cm in length L in the form

$$Nu_L = 0.31Ra_L^{1/4} \qquad 10^5 < Ra_L < 10^{10}. \tag{1}$$

(There has been confusion in the literature regarding the characteristic length L in the Nusselt and Rayleigh numbers. The original Saunders and Fishenden (1950) correlation was based on the square plate half-side-length. However, the standard textbooks quote this correlation as based on the total side length. In this work the length L is taken to be the total side length and the cited correlations are adjusted accordingly.)

Kadambi and Drake [4] recorrelated the original Saunders, Fishenden and Mansion data based upon laminar boundary layer theory indicating a $\frac{1}{5}$ power dependency

$$Nu_L = 0.816Ra_L^{1/5}. \tag{2}$$

Birkebak and Abdulkadir [5] presented results for a 19 cm square plate in water. Three data points were correlated as

$$Nu_L = 0.90Ra_L^{1/5} \qquad 4 \times 10^8 < Ra_L < 8 \times 10^8. \tag{3}$$

Hassan and Mohamed [6] made measurements at angles of tilt ranging from −90° to +90° from the vertical using a 50.4 cm × 20 cm rectangular plate in air. Based upon their results in the central portion of their plate (five data points), they proposed a correlation for an indefinitely large downward-facing horizontal plate (−90°)

$$Nu_L = 0.068Ra_L^{1/3}. \tag{4}$$

To compare theoretical and experimental results, Aihara, Yamada and Endō [7] measured the local velocity and temperature fields near a 25 cm × 35 cm rectangular downward-facing plate in air. Glass plates were aligned against opposite sides of the plate to insure quasi-two-dimensional free convective heat transfer. Average Nusselt numbers were presented as

$$Nu_L = 0.66 Ra_L^{1/5} \qquad Ra_L = 7 \times 10^6$$
$$Nu_L = 0.67 Ra_L^{1/5} \qquad Ra_L = 10^7. \tag{5}$$

Theoretical analysis on free convection from a horizontal plate was initiated by Stewartson [8]. A similarity solution to the boundary layer on a downward-facing semi-infinite horizontal plate was presented. A sign error in the analysis, pointed out by Gill, Zeh and del Casal [9], indicates that Stewartson's result is actually for an upward-facing heated plate and that a similarity solution does not exist for the downward-facing heated plate.

To circumvent this mathematical difficulty Singh, Birkebak and Drake [10] analyzed approximately the natural convection using an integral boundary layer approach with zero boundary layer thickness at the edges. For the square plate they proposed

$$Nu_L = 0.94 Ra_L^{1/5}. \tag{6}$$

Singh and Birkebak [11] revised the analysis by allowing there to be a finite boundary layer thickness at the plate edges.

Clifton and Chapman [12] also analyzed the natural convection from a heated downward-facing infinite strip by the integral boundary layer approach. The boundary layer thickness at the plate edges was set equal to a critical depth obtained from hydraulic flow theory. For fluids with Prandtl number near unity the Nusselt number was given as

$$Nu_L = 0.58 Ra_L^{1/5}. \tag{7}$$

It became the objective of the work reported here to investigate the downward-facing rectangular and square horizontal plates in air, water and a high Prandtl number oil. In particular the effect of adiabatic extensions of various lengths L_a upon the edge boundary layer thickness was thought likely to be important. The results presented here show that the plate aspect ratio L/W and edge effect are significant, and a correlation based upon displacing the boundary layer to zero thickness at an extrapolation length L_e is put forward.

2. APPARATUS AND PROCEDURE

Two experimental test plates were constructed. One plate was fabricated from two 3 mm thick nickel-plated copper sheets 25.4 cm square sandwiching a printed-circuit electrical heater. Another plate was fabricated from a 1 cm thick, 10 cm × 30 cm rectangular, polished aluminum plate. Thermocouples installed in each plate assembly permitted monitoring of the plate temperatures which were esentially isothermal.

Tests in air were made with a laser-holographic interferometer. Following essentially Hauf and Grigull [13], a 20 mW helium–neon laser beam (Spectra-Physics model 124A) was split with a variable half-silvered mirror and expanded from focal points at

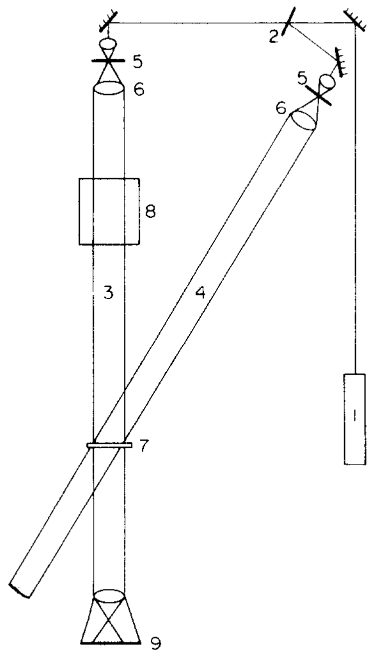

FIG. 1. Experimental arrangement of holographic interferometer. 1 helium–neon laser; 2 beam splitter; 3 object beam; 4 reference beam; 5 spatial filter; 6 collimator; 7 holographic plate; 8 test cell; 9 camera.

micrometer-mounted pinholes by lenses. One of the resulting two 9 cm beams passed over the test plate parallel to it as shown in Fig. 1. The other reference beam crossed the first at a holographic plate holder. With the unheated plate in position, a photographic plate is exposed where the two beams cross, creating a hologram. An image of the test plate is formed by viewing the holographic plate down the test beam axis through a camera lens, after the photographic plate has been developed *in situ*. This image displays a neutral fringe field, and, upon heating the plate, fringes emerge (in real time). Each dark fringe records the locus of a half-wave phase shift (that is, an odd integer times a half wave) along the test beam, and each bright fringe is the locus of a full wave phase shift. The phase shift is linearly related to the average temperature difference above ambient, along the test beam. Hence the temperature field $T(x, y, z)$ causes a fringe where $\bar{T}(x, z)$ is constant, where z is normal to the plate and

$$\bar{T}(x,z) = \frac{1}{W}\int_0^W T(x,y,z)\,\mathrm{d}y. \tag{8}$$

From knowledge of $\bar{T}(x, z)$, the temperature gradient $\partial\bar{T}/\partial z$ can be determined, and the average heat flux is

$$\bar{q} = \frac{1}{L}\int_0^L -k\frac{\partial\bar{T}}{\partial z}\mathrm{d}x = \frac{1}{LW}\int_0^L\int_0^W -k\frac{\partial T}{\partial z}\mathrm{d}y\,\mathrm{d}x. \tag{9}$$

Tests in water and oil were made by calorimetry. Each plate was mounted to a 10 cm thick block of

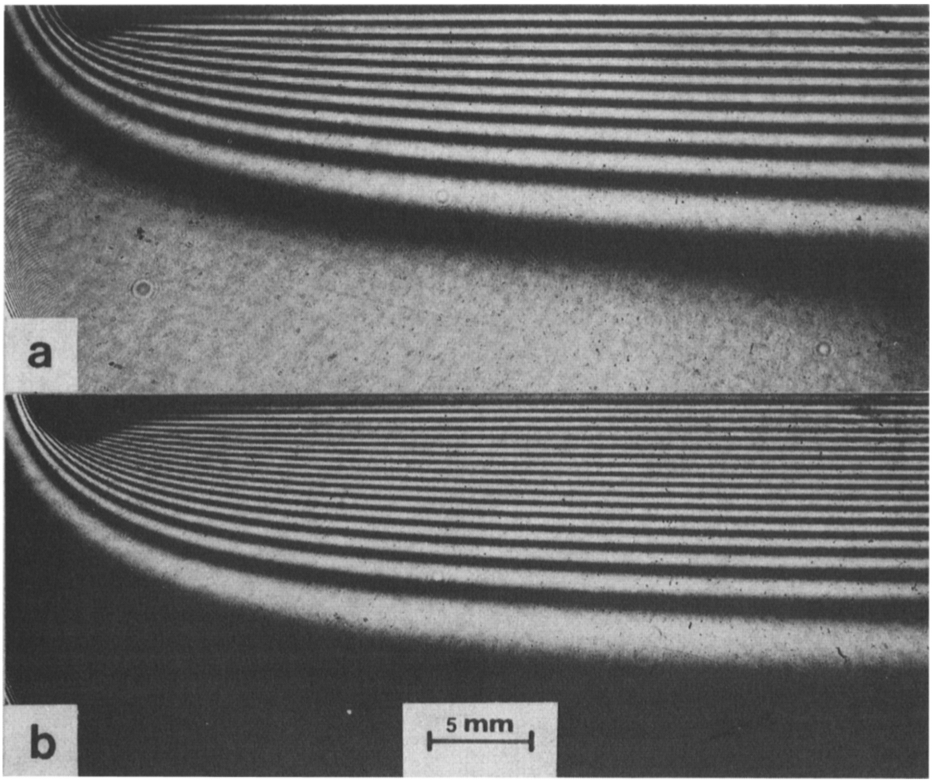

FIG. 2. Thermal boundary layer below 10 cm × 30 cm horizontal heated plate. $L_a/L = 0$. (a) $Ra_L = 3.33 \times 10^6$; (b) $Ra_L = 5.07 \times 10^6$.

polyisocyanurate foam insulation. Heat transfer from the downward-facing side of the heated plate was determined from the measured power input to the electrical heater corrected for conduction heat losses through the foam insulation. The water experiments were carried out in a 48 cm × 53 cm, 22 cm deep tank. The test assemblies were mounted so that the bottom of the plates were 10 cm from the bottom of the tank. To prevent thermal stratification in the water, it was necessary to feed fresh water at the bottom of the tank under a perforated plate while removing warm water near the liquid surface. An optimum feed rate of 1300 cm^3/min was found which was low enough so not to create a free-forced convection situation but high enough to limit stratification to 0.1°C/cm. The tests in oil were performed in a 25 cm × 50 cm, 25 cm deep glass tank. Thermal stratification was limited to 0.3°C/cm by running water-cooled copper tubing through the oil near the top of the tank.

Polystyrene foam extensions up to 50 mm were added to the plates. For the air tests, the foam was faced with first-surface aluminized paper to reduce radiative transfer. Care was taken to align the plate and extension surfaces.

3. RESULTS

Figures 2 and 3 show selected interferograms for the rectangular plate in air. The laser beam is parallel to the long side of the plate. Note that the temperature gradient normal to the wall is almost uniform (purely conductive) out to nearly one-third of the boundary layer thickness in the central portion of the plate, which is a consequence of low tangential velocities there. The uniform gradient eases the determination of the local wall heat flux (averaged over y), because, with the apparatus used, the locations of the first fringe or two adjacent to the heated surface are made somewhat uncertain by diffraction effects. The temperature difference from the center of one bright fringe to another is approximately 2.6°C in the records (the actual temperature difference per fringe goes like the square of the absolute temperature).

Figures 2(a) and 2(b) show the thermal boundary layers of the unextended plates at two different ΔTs. Notice that the turning radius represented by the outer isotherm at the plate edge is only slightly affected by the increase in Rayleigh number. In the central region, the boundary layer thickness is seen to decrease perceptibly (by 8% as Ra goes from 3.3×10^6 to 5.1×10^6), in accord with boundary layer theory. To see the decrease observe the thickness over which say 75% of the temperature difference occurs. The variation in boundary layer thickness in distance from the plate edge is in agreement with integral-method boundary layer theory [11]. For instance, the ratios of the edge-

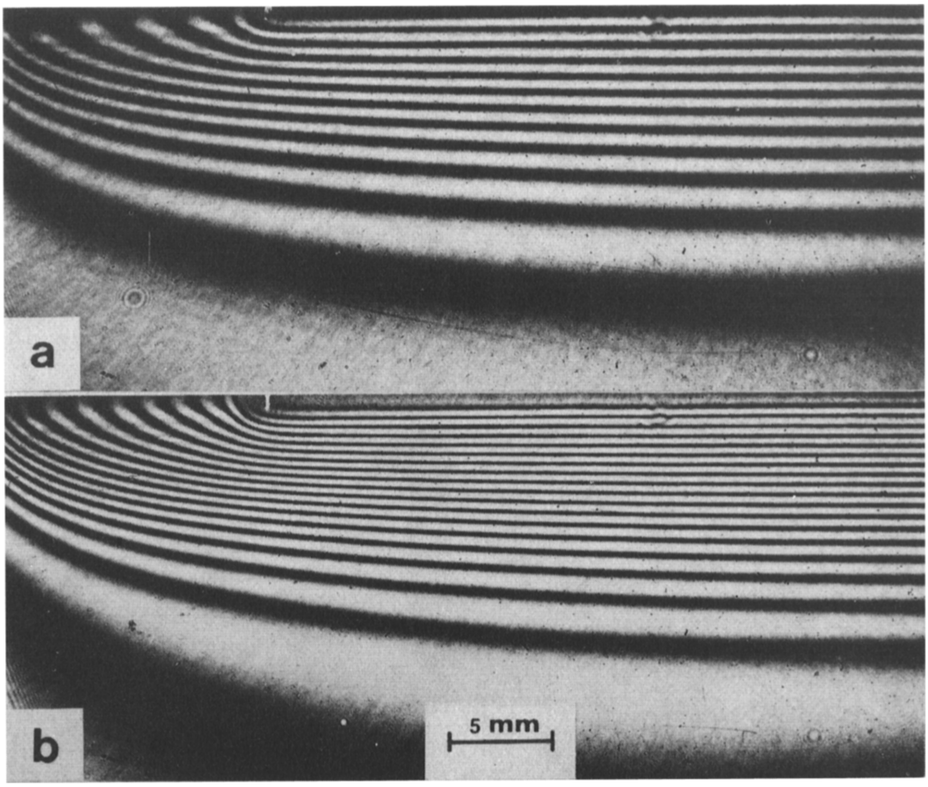

FIG. 3. Thermal boundary layer below 10 cm × 30 cm horizontal heated plate. $L_a/L = 0.125$. (a) $Ra_L = 3.33 \times 10^6$; (b) $Ra_L = 5.07 \times 10^6$. (Vertical lines locate plate-extension junctions.)

to-center boundary layer thickness is 0.50 in the records [Figs. 2(a) and 2(b)] whereas theory [11] predicts 0.53.

Figures 2 and 3 show the large increase in boundary layer thickness at the edge when a 12.5 mm (L_a/L = 0.125) adiabatic extension is present.

The present air results are plotted in Fig. 4 as

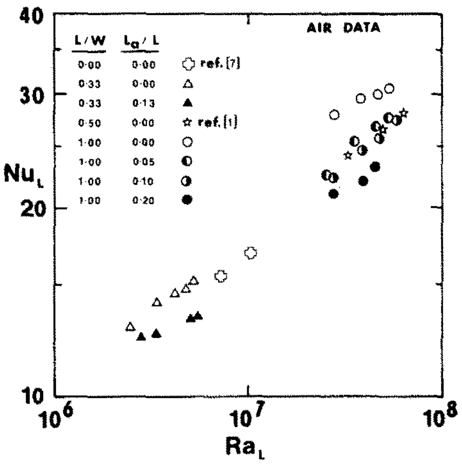

FIG. 4. Experimental data for downward-facing horizontal heated plates in air.

Nusselt number vs Rayleigh number. Also shown are the results of Saunders, Fishenden and Mansion [1] for a 23 cm × 46 cm (L/W = 0.50) plate in air and the results of Aihara, Yamada and Endō [7] for a 25 cm × 35 cm plate in air. In the latter study the data are taken to be representative of an infinite strip ($L/W = 0$) since vertical glass panes were aligned against opposite sides of the plate. The effect of plate aspect ratio L/W is seen to be significant by comparing the present square plate results (open circles in the figure) to the Saunders, Fishenden and Mansion rectangular plate results (stars). The reduction in Nusselt number due to addition of adiabatic extensions is readily apparent.

4. CORRELATION

An approximate correlation form is proposed on the basis of displacing the origin of a 2-D boundary layer by extrapolation length L_e. Boundary layer theory [10] has Nu_x going as $Ra_x^{1/5}$. However, to hold the option open for choosing another slope to better fit data, we take Nu_x to go as Ra_x^m. Then the average heat transfer coefficient and Nusselt number become

$$\bar{h} = \frac{2}{L}\int_{L_e}^{L/2+L_e} h(x)\,dx \tag{10}$$

$$Nu_L = \frac{\bar{h}L}{k} = C_1 Ra_L^m\left[\left(1+\frac{2L_e}{L}\right)^{3m} - \left(\frac{2L_e}{L}\right)^{3m}\right]. \tag{11}$$

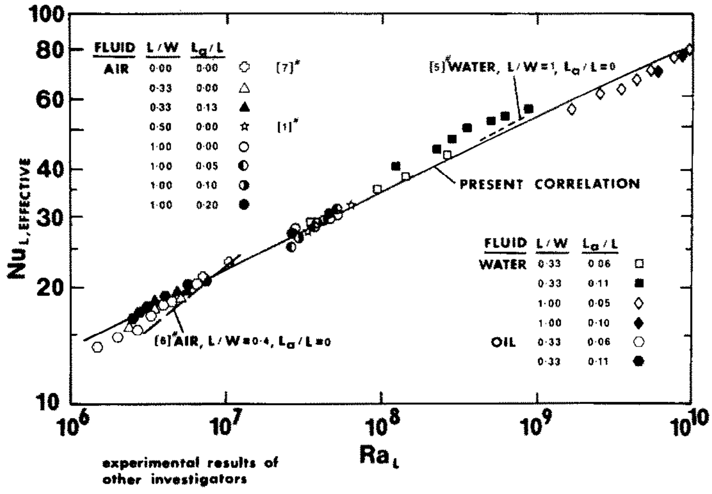

FIG. 5. Comparison of experimental data with correlation for downward-facing horizontal heated plates.

The dimensionless length $2L_e/L$ for the bare edge is taken to scale as $C_2Ra_L^n$. In the case of an adiabatic extension, a term $C_3(L_a/L)^p$ is added. Finally an aspect ratio correction for $L/W(\leq 1)$ is taken. Thus the correlation form proposed is

$$Nu_L = C_1[1+C_4L/W][(1+X)^{3m} - X^{3m}]Ra_L^m \quad (12)$$

$$X = C_2Ra_L^n + C_3(L_a/L)^p. \quad (13)$$

The seven coefficients in the correlation were chosen to fit the air data in Fig. 4. The present bare-edge square plate and Saunders, Fishenden and Mansion [1] rectangular plate results fix C_2, and the present bare-edge square and rectangular plate local Nusselt number data fix C_1, m, C_2 and n. Finally C_3 and p are chosen to correlate the data for the adiabatic extensions. The correlation for $L/W = 1$ and $L_a/L = 0$ (bare-edge square plate) is shown in Fig. 5 with

$$C_1 = 6.5 \quad C_2 = 0.38 \quad C_3 = 13.5 \quad C_4 = 2.2$$

$$m = 0.13 \quad n = -0.16 \quad p = 0.7.$$

The experimental results in Fig. 5 are presented as effective bare-edge square plate Nusselt numbers via equations (12) and (13). Plotted is $Nu_{L,\text{effective}}$ vs Ra_L where

$$Nu_{L,\text{effective}} = Nu_{L,\text{exp}} \times \frac{Nu_L(Ra_L, L/W = 1, L_a/L = 0)}{Nu_L(Ra_L, L/W, L_a/L)} \quad (14)$$

and $Nu_L(Ra_L, L/W, L_a/L)$ is given by equation (12). The Hassan and Mohamed [6] correlation, shown in the figure, required an additional correction since the correlation was based on the local Nusselt number at the center of a rectangular plate. This correction was derived from our interferograms to account for the edge effects; it amounted to an increase of 30%. Although the coefficients in the correlation were chosen to fit the air data, the present water (Pr = 6) and oil (Pr = 4800) results and the Birkebak and Abdulkadir [5] experimental results for a square plate in water are correlated as well. The relative insensitivity of Nusselt number to Prandtl number greater than unity, except through Rayleigh number, is in agreement with boundary layer theory [11, 14].

The proposed correlation fits the present data and those of other investigators to within ±10%. The data correlated include aspect ratios from zero (infinite strip) to unity (square plate), Prandtl number from 0·7 to 4800, and adiabatic extensions with La/L up to 0.2.

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REFERENCES

1. O. A. Saunders, M. Fishenden and H. D. Mansion, Some measurements of convection by an optical method, *Engineering* **139**, 483 (1935).
2. R. Weise, Warmeubergang durch freie Konvektion an quadratischen Platten, *Forsch. Geb. Ing.-Wes.* **6**, 281 (1935).
3. M. Fishenden and O. A. Saunders, *An Introduction to Heat Transfer*, p. 96. Oxford University Press, London (1961).
4. V. Kadambi and R. M. Drake, Free convection heat transfer from horizontal surfaces for prescribed variations in surface temperature and mass flow through the surface. Technical report Mech. Eng. HT-1, Princeton University (1960).
5. R. C. Birkebak and A. Abdulkadir, Heat transfer by natural convection from the lower side of finite horizontal, heated surface, 4, NC2.2 Preprints, *4th Int. Heat Transfer Conf.*, Paris (1970).
6. K. E. Hassan and S. A. Mohamed, Natural convection from isothermal flat surfaces, *Int. J. Heat Mass Transfer* **13**, 1873 (1970).
7. T. Aihara, Y. Yamada and S. Endō, Free convection

along the downward-facing surface of a heated horizontal plate, *Int. J. Heat Mass Transfer* **15**, 2535 (1972).
8. K. Stewartson, On the free convection from a horizontal plate, *Z. Angew. Math. Phys.* **9**, 276 (1958).
9. W. N. Gill, D. W. Zeh and E. del Casal, Free convection on a horizontal plate, *Z. Angew. Math. Phys.* **16**, 539 (1965).
10. S. N. Singh, R. C. Birkebak and R. M. Drake, Laminar free convection heat transfer from downward-facing horizontal surfaces of finite dimensions, *Progress in Heat and Mass Transfer*, (T. F. Irvine, editor). Pergamon Press, Oxford (1969).
11. S. N. Singh and R. C. Birkebak, Laminar free convection from a horizontal infinite strip facing downwards, *Z. Angew. Math. Phys.* **20**, 454 (1969).
12. J. V. Clifton and A. J. Chapman, Natural convection on a finite-size horizontal plate, *Int. J. Heat Mass Transfer* **12**, 1573 (1969).
13. W. Hauf and U. Grigull, Optical methods in heat transfer, *Advances in Heat Transfer* **6**, (J. P. Hartnett, editor). Academic Press (1970).
14. Z. Rotem and L. Claassen, Free convection boundary layer flow over horizontal plates and discs, *Can. J. Chem. Engng* **47**, 461 (1969).

EFFETS DE BORD ET DE RAPPORT DE FORME SUR LA CONVECTION NATURELLE D'UNE PLAQUE CHAUDE, HORIZONTALE, TOURNEE VERS LE BAS

Résumé—On donne des résultats d'une étude expérimentale sur la convection naturelle d'une plaque chaude, horizontale, tournée vers le bas. Des mesures sont effectuées avec des plaques carrées et rectangulaires dans l'air, l'eau et dans une huile à nombre de Prandtl élevé. Les plaques ont des bords nus ou des extensions approximativement adiabatiques autour des bords. Une explication et une formulation des effets de bord sont faites à partir du déplacement de l'origine d'une solution de couche limite. La formulation ainsi obtenue tient compte des bords nus ou des extensions adiabatiques, différemment des formulations antérieures.

EINFLÜSSE VON KANTEN- UND SEITENVERHÄLTNISSEN AUF DIE NATÜRLICHE KONVEKTION AN EINER HORIZONTALEN, AN IHRER UNTERSEITE BEHEIZTEN PLATTE

Zusammenfassung—Es werden Ergebnisse einer experimentellen Untersuchung über natürliche Konvektion an einer horizontalen, nach unten Wärme abgebenden Platte mitgeteilt. Die Messungen wurden an quadratischen und rechteckigen Platten in Luft, Wasser und einem Öl mit hoher Prandtl-Zahl durchgeführt. Die Platten wurden mit bloßen Kanten und mit nahezu adiabaten Fortsetzungen an den Kanten ausgeführt. Durch Verschieben des Ursprungs einer Grenzschichtlösung wird eine Erklärung und Korrelation der Kanteneinflüsse durchgeführt. Die so erhaltene Korrelation berücksichtigt sowohl bloße Kanten als auch adiabate Fortsetzungen und Seitenverhältnisse, wie sie bislang noch nicht untersucht wurden.

ВЛИЯНИЕ КОНЦЕВЫХ ЭФФЕКТОВ И ОТНОШЕНИЯ СТОРОН НА ЕСТЕСТВЕННУЮ КОНВЕКЦИЮ ОТ ГОРИЗОНТАЛЬНОЙ ПЛАСТИНЫ, ОБРАЩЕННОЙ НАГРЕТОЙ СТОРОНОЙ ВНИЗ

Аннотация — Представлены результаты экспериментального исследования естественной конвекции от горизонтальной пластины, обращенной нагретой стороной вниз. Пластины изготавливались в форме квадратов и прямоугольников и помещались в воздух, воду и масло, характеризующееся большим числом Прандтля. Использовались пластины как со свободными краями, так и с адиабатическими приставками вокруг краев. Путем смещения начала координат в решении пограничного слоя дано объяснение концевых эффектов и выведена обобщающая формула. В полученной таким образом зависимости, по сравнению с ранее полученными соотношениями, учитывается влияние свободных краев и адиабатических приставок, а также отношения сторон пластин.